Newton's Laws: Jumping Grasshoppers 1

Figure 1: *Schistocerca americana* (American bird grasshopper)

Introduction

Many kinds of creatures jump. That is, they move by pushing off the ground and moving through the air both horizontally and vertically. Their motion is determined by the contact force exerted on them by the ground (while they are still in contact with it) and by the weight force exerted by the Earth. For an example, watch the video of a jumping grasshopper at https://www.youtube.com/watch?v=qhvlCErCZE0. In this activity, you will explore those forces and how they affect the motion of *Schistocerca americana* (the American grasshopper found throughout the Southeast), as reported in a 1995 publication[1] by two biologists from the University of California at Berkeley.

These scientists measured the distance grasshoppers jumped on various days during their sixth instar[2] as well as their masses and the forces they exerted on the ground. To measure the forces, the scientists built a miniature platform from model airplane plywood and mounted it on leaf springs with semiconductor strain gauges[3] affixed. When they placed an insect on the platform and induced it to jump by startling it, the strain gauges told them how much force the insect exerted on the platform as it was jumping.

Learning Goals

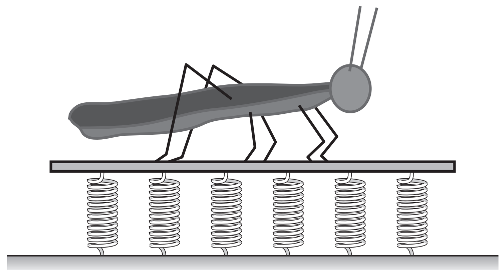
After completing this activity, you should be able to…

- Numerically integrate an acceleration versus time graph and velocity versus time graph in order to determine the change in velocity and change in position of an object respectively.
- Demonstrate proficiency in solving problems in dynamical contexts by applying Newton's laws.

1. E.J. Queatheman and R.J. Full, "Variation in jump force production within an instar of the grasshopper *Schistocerca americana*," certain *J. Zoology* **235**, 605-620 (1995).
2. An instar is the period between successive moults.
3. The electrical properties of materials change in a predictable way when those materials are compressed by an applied force. This allows you to use the amount of electrical current flowing through a bit of that material to measure the force that is being applied to it.

A. Force and acceleration of a jumping grasshopper in one dimension

Consider the grasshopper at rest on the scale as shown.



1. Draw a free-body diagram for the grasshopper. The lengths of the arrows should be consistent with the relative magnitude of the forces.
2. How do the forces exerted on the grasshopper compare in magnitude? Explain.

Now consider the case when the grasshopper begins to jump. To keep the analysis one-dimensional, assume the grasshopper jumps straight up.

3. As the grasshopper moves from rest to an upward velocity, does the change in velocity vector point upward, downward, or is it zero? Explain.
4. What is the direction of the grasshopper's acceleration and the net force it experiences during the jump? Explain.
5. Draw a free-body diagram for the grasshopper during the initial stages of the jump. Note that the grasshopper is still in contact with the scale. The lengths of the arrows should be consistent with the relative magnitude of the forces.
6. Two students are discussing their answers to question A5:

   David: *The free-body diagram for question 5 looks exactly the same as that for question 2. The weight can't change since the mass stayed the same. The grasshopper is still in contact with the scale, so the force by the scale cannot change.*

   Colin: *You're right that the weight must be the same. But for the free-body diagram in question 5, the force that the scale exerts must be bigger than the weight force in order for the grasshopper to experience a net upward force.*

   Do you agree or disagree with either of the students? Explain your reasoning.

✓ Check with an instructor before proceeding.

Now consider the data reported in the publication. The figure below shows the force that the grasshopper exerted on the scale as a function of time.

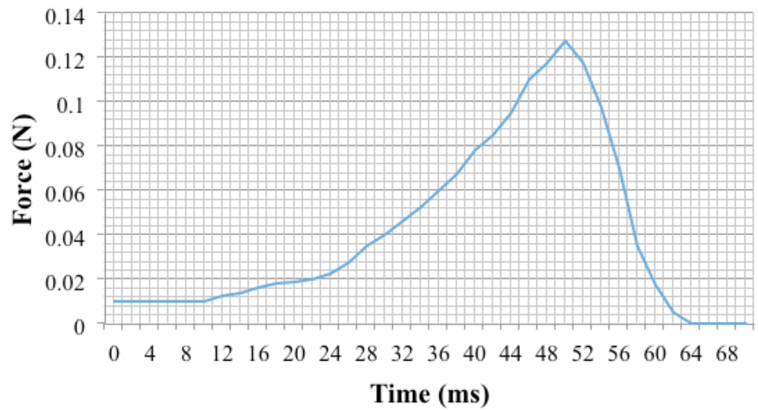


Figure 2: Force exerted by the grasshopper on the platform versus time. The data are tabulated in the Excel file available for the studio on Sakai.

7. Does the force that the grasshopper exerts on the scale point in the same or opposite direction to the force the scale exerts on the grasshopper?
8. Is the magnitude of the force exerted by the grasshopper on the scale greater than, less than, or equal to the magnitude of the force exerted by the scale on the grasshopper at $t = 0$ ms? Explain.
9. Repeat question A8 for t = 36 ms, t = 50 ms, and t = 56 ms.
10. Even though the graph in Figure 2 is showing the force that the grasshopper exerts on the scale as a function of time, explain why it is reasonable for us to use the data to represent the force that the scale exerts on the grasshopper.
11. Make a sketch of the graph shown in Figure 2. Label the following events on the force versus time graph. For each event, if you don't think that the corresponding time is represented on the graph, state so explicitly. Explain your reasoning for each answer.
    a. Put an "A" on the curve at the instant when the grasshopper begins to jump.
    b. Put a "B" on the curve at the instant when the grasshopper completely loses contact with the scale.
    c. Put a "C" on the curve at the instant when the grasshopper has the greatest acceleration away from the scale.

12. Two students are discussing their answers to Question A11.

    Omar: *I think the grasshopper jumps off the plate at 10 ms. The grasshopper keeps going upward until the graph is at its maximum at 50 ms. This is the instant when the grasshopper is at its maximum height above the plate. The grasshopper is falling back to the plate during the time interval when the slope of the graph is negative, between 50 ms and 64 ms. It lands on the plate at 64 ms and stays there for the rest of the time shown on the graph.*

    Natasha: *The graph shows the force the grasshopper exerts on the plate, not its height above the plate. The grasshopper begins to jump at 10 ms. The force it exerts on the plate increases until 50 ms, so that's when the grasshopper is accelerating up the most. Then the force decreases again until the grasshopper loses contact with the plate at 64 ms, so the grasshopper speeds up less during that time. After 64 ms, the grasshopper continues to move upward, but the downward force of gravity will be gradually slowing it down.*

    Do you agree or disagree with either or both of the students? Explain your reasoning.

13. Sketch a graph of the *net force* applied to the grasshopper as a function of time.

14. Determine the mass of the grasshopper.

15. Please download and open the Excel file template that contains the data for Figure 2.

    a. For each instant shown, use Excel to calculate the net force on the grasshopper. Insert your values in the column labeled "$F_{\text{net,g}}$ (N)."

    b. Plot the net force versus time in Excel, using the data you just calculated. Does the plot generated by Excel look similar to the one you sketched for question A13? If not, explain what went wrong.

    c. Next to the net force column you just completed, there is a column for the acceleration experienced by the grasshopper at various times. For each instant shown, use Excel to calculate the acceleration.

    d. In Excel, plot the acceleration of the grasshopper as a function of time.

B. Deducing velocity from acceleration

In this section, we will use the grasshopper's acceleration as a function of time to determine its velocity as a function of time.

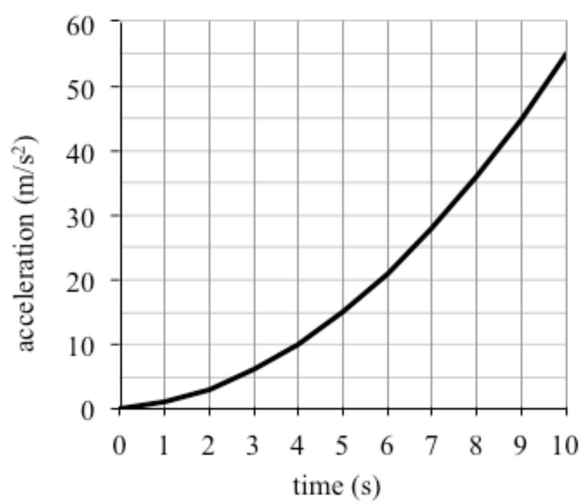
1. Make a qualitatively correct sketch of the velocity versus time plot (*i.e.*, don't worry about the precise velocities involved – just try to get the basic shape right).

The change in velocity over a time interval can always be found by calculating the area under the curve of an acceleration versus time graph. However, when the acceleration changes with time, it can be difficult to calculate the exact value of the area under the curve. A method called numerical integration allows us to approximate the area under the curve. If you want more information about numerical integration, please visit http://people.oregonstate.edu/~haggertr/487/integrate.htm

2. In class, you encountered the relationship, $a = \Delta v/\Delta t$, which can be rewritten as $\Delta v = a \cdot \Delta t$. Explain what has to be true about the acceleration, a, during the time interval Δt for this relationship to correctly describe the change in velocity over that same time interval.

To numerically integrate, we assume that the acceleration has a constant value for each time interval.

Three students are trying three different numerical integration techniques to approximate the area under the curve of the acceleration versus time graph at right.



- Andrew uses the value of the acceleration at the beginning of each time interval as the acceleration for the entire time interval (*e.g.*, he uses 10 m/s^2 as the acceleration from 4 s to 5 s).
- Carla uses the value of the acceleration at the end of each time interval as the acceleration for the entire time interval (*e.g.*, she uses 15 m/s^2 as the acceleration from 4 s to 5 s.).
- Jessica uses the average value of the acceleration within each time interval as the acceleration for the entire time interval (*e.g.*, she uses 12.5 m/s^2 as the acceleration from 4 s to 5 s.).

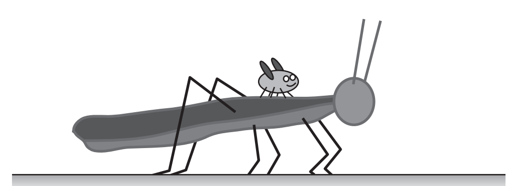
3. For each student, draw a graph that illustrates their technique by sketching in the rectangles that represent the areas they are calculating for each 1 s time interval.
4. For each student, state whether they over- or underestimate the area under the curve. If you cannot tell, state so explicitly. Explain your reasoning.
5. Which student's numerical integration technique will give the best approximation to the change in velocity from 0 s to 10 s? Explain.
6. Work with your group members to create a formula in Excel that allows you to best approximate the change in velocity of the grasshopper in each time interval shown in column F. Enter the forumula in the spreadsheet in the column "Δv (m/s)", and populate the entire column.

✓ Check with an instructor before proceeding.

7. The next step is to populate the column "v(t)." As a beginning point, suppose that you want to know the speed of the grasshopper at 2 ms. What pieces of information would your Excel formula need to contain?
8. Create a formula in Excel that allows you to calculate the velocity at each instant. Enter your formula into column "v(t)," and populate the entire column. Use the values in the "v" column to plot the velocity of the grasshopper as a function of time.
9. How does your graph compare to your prediction in question B1? Briefly explain.
10. Upload your Excel spreadsheet and make sure to save a copy as you may need to use the same spreadsheet in the next studio.

C. Applications and Extensions

To assess your understandings of some of this studio's key ideas, your group must answer the following questions together without help from the instructors or other groups.



Suppose that a small bug sits on top of the grasshopper as it begins to jump. The mass of the bug is less than that of the grasshopper. Consider an interval of time during which the grasshopper is jumping vertically upwards and is speeding up. You can ignore air resistance.

1. For this time interval, draw free-body diagrams for the bug and for the grasshopper.
2. Identify any third law force pairs in your diagrams.
3. Rank all the forces exerted on the bug and grasshopper by magnitude, from largest to smallest, and explain your reasoning.
4. Suppose the grasshopper's mass, *m,* is five times that of the small bug. In terms of *mg* (the weight force on the grasshopper), determine the magnitude of all other forces.